Tea: Cover's Guessing Game

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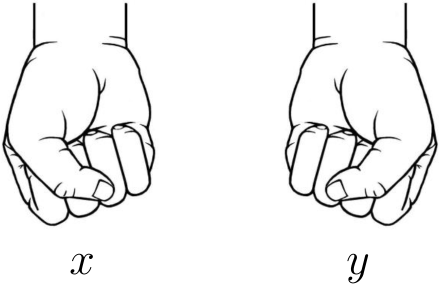


THEM: Choose left or right.

YOU: Left?

THEM: *Reveals* $x = \pi$.

THEM: Is $y > x$ or not?

Can you win with probability strictly greater than $\frac{1}{2}$?

Yes, You Can Win With Probability $> \frac{1}{2}$!

- Choose a function $p\colon \mathbb{R} \to [0,1]$ that is strictly increasing.
- Strategy:
  1. Choose to see $z \in \{x, y\}$ with equal probability.
  2. Guess other is lower with probability $p(z)$.
- Proof:
  - Let $H = \max(x, y)$ and $L = \min(x, y)$.
  - Win probability equal to

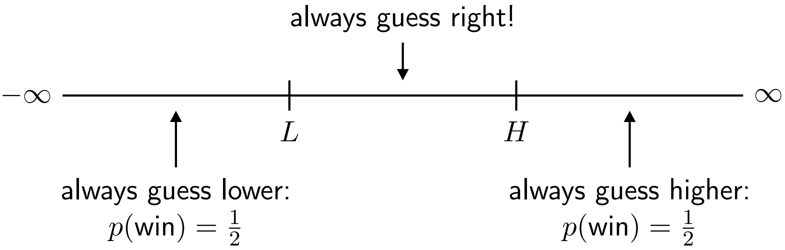
$$\frac{1}{2}p(H) + \frac{1}{2}(1 - p(L)) = \frac{1}{2} + \frac{1}{2}(p(H) - p(L)) > \frac{1}{2}.$$

↑ you happen to see H

- What is this madness?!?!

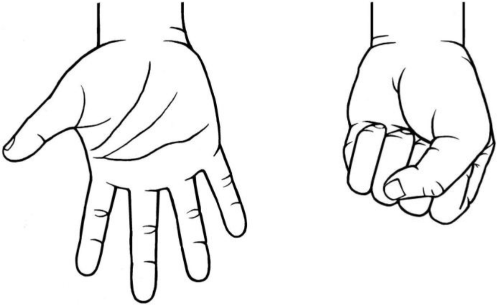
Another Strategy: Random Threshold

- Strategy:
  1. Draw $r \sim \mathcal{N}(0, 1)$.
  2. Choose to see $z \in \{x, y\}$ with equal probability.
  3. Guess other is lower if $z > r$.



Cover's Game

- Popularised by half-page abstract of Thomas Cover (1987).
- Paradox can be traced back to works of David Blackwell and Bruce Hills (1951!).
- Many related puzzles! See Gnedin (2016).



BANANAN: I myself have invented a game. Well, think of a number.
ALIKA: I got a number.
BANANAN: Me too. Now, tell me yours.
ALIKA: Seven.
BANANAN: Seven. Mine is eight—I won.

Sergey Solovyov, *Assa* (Quote taken from Gnedin (2016).)

These slides: `https://wesselb.github.io/pdf/cover`.

Appendix

References

Cover, T. M. (1987). Pick the largest number (T. M. Cover & B. Gopinath, Eds.; 1st ed.). *Open Problems in Communication and Computation*, 152–152.

Gnedin, A. (2016). Guess the larger number. *arXiv preprint arXiv:1608.01899*.